OBJECTIVE MATHEMATICS

Volume 2

Descriptive Test Series

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CHAPTER-7 : INDEFINITE INTEGRATION

UNIT TEST-1

1. If $f(x)=\int \frac{5x^8+7x^6}{\left(x^2+1+2x^7\right)^2}dx,(x\geq 0),f(0)=0$ and $f(1)=\frac{1}{K}$ then the value of K is _____.

2. If $\int \frac{\sin x}{\sin^3 x+\cos^3 x}dx=\alpha\log_e|1+\tan x|+\beta\log_e\left|1-\tan x+\tan^2 x\right|+\gamma\tan^{-1}\left(\frac{2\tan x-1}{\sqrt{3}}\right)+C$, when C is constant of integration, then the value of $18\left(\alpha+\beta+\gamma^2\right)$ is _____.

Hints and Solutions

1. (04) $\int \frac{5x^8+7^6}{x^{14}\left(\frac{1}{x^5}+\frac{1}{x^7}+2\right)}dx \Rightarrow \int \frac{5x^{-6}+7x^{-8}}{\left(2+\frac{1}{x^5}+\frac{1}{x^7}\right)^2}dx$

Put $2+\frac{1}{x^5}+\frac{1}{x^7}=t$

$\left(-5x^{-6}-7x^{-8}\right)dx=dt \Rightarrow \int\frac{-dt}{t^2}=\frac{1}{t}+c$

$f(x)=\frac{x^7}{2x^7+x^2+1}+c$

$f(0)=0\Rightarrow c=0\Rightarrow f(1)=\frac{1}{4}\Rightarrow k=4$

2. (3) $I=\int\frac{\sin x}{\cos^3 x+\sin^3 x}dx=\int\frac{\frac{\sin x}{\cos^3 x}}{\frac{\cos^3 x}{\cos^3 x}+\frac{\sin^3 x}{\cos^3 x}}\cdot dx$

$I=\int\frac{\tan x\cdot\sec^2 x}{1+\tan^3 x}dx,\quad$ Put $\tan x=t$

$\sec^2 x\cdot dx=dt$

$I=\int\frac{t}{1+t^3}dt$

$\frac{t}{1+t^3}=\frac{A}{1+t}+\frac{Bt+C}{1+t^2-t}$

$t=A\left(1-t+t^2\right)+(1+t)(Bt+C)$

By comparing coeff. of x, x^2 and constant term,

$A=-\frac{1}{3},B=\frac{1}{3},C=\frac{1}{3}$

$I=-\frac{1}{3}\int\frac{1}{1+t}dt+\frac{1}{3}\int\frac{t+1}{t^2-t+1}dt$

$I=-\frac{1}{3}\ln(1+t)+\frac{1}{6}\left[\int\frac{2t-1}{t^2-t+1}dt+3\int\frac{1}{t^2-t+1}dt\right]$

$I=-\frac{1}{3}\ln(1+t)$

$+\frac{1}{6}\left[\log\left(t^2-t+1\right)+3\cdot\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2t-1}{\sqrt{3}}\right)\right]+C$

$I=-\frac{1}{3}\ln(1+\tan x)+\frac{1}{6}\cdot\log\left(\tan^2 x-\tan x+1\right)$

$+\frac{1}{\sqrt{3}}\cdot\tan^{-1}\left(\frac{2\tan x-1}{\sqrt{3}}\right)+C$

$\alpha=-\frac{1}{3},\beta=\frac{1}{6},\gamma=\frac{1}{\sqrt{3}}$

$18\left(\alpha+\beta+\gamma^2\right)$

$18\left(-\frac{1}{3}+\frac{1}{6}+\frac{1}{3}\right)=3$